NJL Model at Finite Chemical Potential in a Constant Magnetic Field *

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Abstract

We investigate the influence of an external magnetic field on chiral symmetry breaking in the Nambu-Jona-Lasinio (NJL) model at finite temperature and chemical potential. According to the Fock-Schwinger proper-time method, we calculate the effective potential in the leading order of the $1/N_{\rm c}$ expansion. The phase boundary dividing the symmetric phase and the broken phase is illustrated numerically. A complex behavior of the phase boundary is found for large chemical potential.

1 Introduction

Strong interactions between quarks and gluons are described by QCD. In resent years much interest has been paid to the phase structure of the QCD vacuum. A chiral $\mathrm{SU}(N)_{\rm L}\times\mathrm{SU}(N)_{\rm R}$ symmetry for quark flavors is broken down by the QCD dynamics. The broken chiral symmetry is restored at

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high temperature and/or high density. One of the most interesting objects may be found in the neutron star which is laid near the critical chemical potential at low temperature. Some of neutron stars have a strong magnetic field. Therefore we investigate the QCD vacuum state at finite temperature and density in a magnetic field.

In the present paper we use the NJL model [1] as one of the simplest low energy effective theory of QCD and evaluate the effective potential in the leading order of $1/N_{\rm c}$ expansion. The Fock-Schwinger proper-time method [2] is applied to a thermal field theory to obtain the exact expression of the effective potential about an external magnetic field. There are several works to study the NJL model at finite temperature and density in an external magnetic field [3, 4]. Here we use a new formula of the two-point function proposed in Ref. [5] where we include the contribution from poles on the complex proper-time plane.

The paper is organized as follows. In the next section we give the explicit expression of the effective potential with involving the temperature T, chemical potential μ and constant magnetic field B in the NJL model. We briefly discuss how to deal with the proper-time integral. Section 3 is devoted the study of phase structure in the NJL model at finite T, μ and B. We give some concluding remarks in the last section.

2 NJL model at finite T, μ and B

NJL model with two flavors is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi}(\imath\partial\!\!\!/ - QA\!\!\!/)\psi + \frac{G}{2N_{\rm c}}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\boldsymbol{\tau}\psi)^2] \ , \tag{1}$$

where $N_{\rm c}$ is the number of colors and G is an effective coupling constant. The quark field ψ belongs to a fundamental representation of the color SU(N) group and a flavor isodoublet. $\boldsymbol{\tau}$ represents the isospin Pauli matrices and Q is the electric charge of the quark fields, $Q = \mathrm{diag}(2e/3, -e/3)$. The Lagrangian is invariant under the global chiral transformation, $\psi \rightarrow \exp[\mathrm{i}\theta\gamma_5\tau^3/2]\psi$. In four space-time dimensions the NJL model is not renormalizable. We regard the theory as a low energy effective theory stemming from a more fundamental theory at a cut-off scale Λ.

For practical calculations it is more convenient to introduce auxiliary

fields σ and π. The Lagrangian (1) is rewritten by using the auxiliary fields

$$\mathcal{L}_{\rm aux} = \bar{\psi}(\imath\partial\!\!\!/ - Q A\!\!\!/)\psi - \bar{\psi}(\sigma + \imath\gamma_5\tau^3\pi)\psi - \frac{N_{\rm c}}{2G}(\sigma^2+\pi^2)\ , \tag{2}$$

where $\sigma \simeq -(G/N_{\rm c})\bar{\psi}\psi$, $\pi = \pi^3 \simeq -(G/N_{\rm c})\bar{\psi}\imath\gamma_5\tau^3\psi$ and we assume that the ground state dose not break the local $\mathrm{U}(1)_{\rm EM}$ symmetry, i.e. $\langle\pi^{\pm}\rangle = 0$. Here we consider a thermal equilibrium state in an external magnetic field and introduce the temperature in according with the imaginary-time formalism. For a conserved charge of $\psi \to \exp[\mathrm{i}\theta\tau^3/2]\psi$ invariance we define a quark chemical potential μ and we assume $\mu_u = \mu_d$.

To find a ground state of the system at finite temperature and chemical potential in an external magnetic field we evaluate an effective potential. In the leading order of the $1/N_{\rm c}$ expansion the effective potential $V(\sigma, \pi = 0)$ is given by

$$V(\sigma, \pi=0) = \frac{1}{4G}\sigma^2 - \frac{1}{2\int_0^\beta \mathrm{d}^4x}\mathrm{Tr}\ \ln(\imath\partial\!\!\!/ + Q A\!\!\!/ - \sigma - \imath\gamma_4\mu)\ , \tag{3}$$

where $\beta = 1/T$. Since the ground state is an isospin singlet, we set $\pi = 0$. The ground state of the thermal average of σ is given by the minimum of the effective potential. If σ develops a non-vanishing value of the ground state, the chiral symmetry is broken down dynamically.

To calculate the effective potential in a strong magnetic field we use the Fock-Schwinger proper-time method. The second term of the right hand side in Eq. (3) is rewritten as

$$\mathrm{Tr}\ \ln(\imath\partial\!\!\!/ + Q A\!\!\!/ - \imath\mu\gamma_4 - \sigma) \simeq -\mathrm{Tr}\int_0^\sigma \mathrm{d}m\ S(x,x;m)\ , \tag{4}$$

$$(\imath\partial\!\!\!/ + Q A\!\!\!/ - \imath\mu\gamma_4 - m)S(x,x';m) = \delta^4(x-x')\ . \tag{5}$$

In the present paper we consider the constant magnetic field, $A_\mu(x) = \delta_{\mu 2}x_1 B$, for simplicity. The explicit expression of the Green function S is described in Ref. [5].

Substituting the Green function $S(x,x;m)$ to Eq. (4) and performing the integration about m, we obtain the second term of the right hand side in Eq. (3). Therefore the effective potential reads

$$\begin{aligned} V(\sigma) \ = \ & \frac{1}{4G}\sigma^2 + \left\{ \frac{\mathrm{e}^{-3\imath\pi/4}}{4\pi^{3/2}\beta}\sum_{n=0}^{\infty}\int_{C_1}\mathrm{d}\tau \right. \\ & \left. \times\frac{QB\tau}{\tau^{5/2}}\cot(QB\tau)\mathrm{e}^{-\imath(\omega_n - \imath\mu)^2\tau}(\mathrm{e}^{-\imath\sigma^2\tau} - 1) + (\mathrm{c.c.}) \right\}\ . \end{aligned} \tag{6}$$

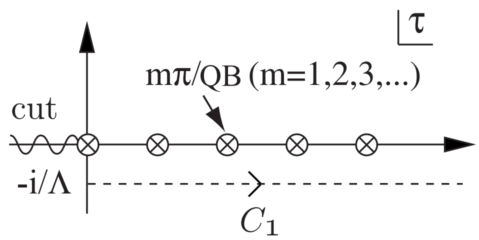


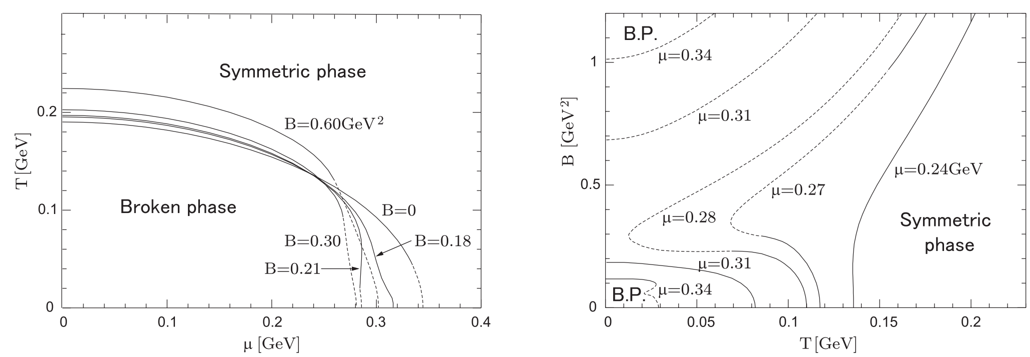
Figure 1: The contour of the integration in Eq. (6). The circles denote the poles.

It should be noted that we must pay attention to a contour of the proper-time integral. The physical contour is shown as a dashed line in Fig. 1.

3 Phase structure

We perform the summation and the integration in Eq. (6) numerically. The coupling constant G and the proper-time cut-off Λ are determined to reproduce a pion decay constant (93MeV) and a pion mass (138MeV). The value of G and Λ depends on the regularization procedure. In the proper-time cut-off regularization the parameters G and Λ are determined to be $G = 38.7\text{GeV}^{-2}$ and $\Lambda = 0.864\text{GeV}$ with accounting for the current quark mass, $m_u + m_d = 14\text{MeV}$, see Ref. [6]. We neglect the contribution from the current quark mass to the effective potential. Evaluating the minimum of the effective potential we find the critical value of T, μ and B.

Figure 2.(a) shows critical lines on the $T - \mu$ plane with B fixed. At $B = 0$ the critical T and μ are a little bit higher than a known value in a momentum cut-off regularization, e.g. $T_{\text{cr}}|_{\mu=0} \simeq 0.17\text{GeV}$ [7]. For small μ a broken phase spreads out uniformly as B increases. For large μ more complex situation is observed. The broken phase reduces one time near $B \sim 0.2\text{GeV}^2$. As is seen in Fig. 2.(b) the broken phase is separated into two parts for $\mu > 0.28\text{GeV}$. A distortion of critical curve at $\mu = 0.34$ GeV is caused by the de Haas-van Alphen effect [8, 9]. From these figures we see that the magnetic field enhances the dynamical symmetry breaking below a certain chemical potential, this phenomenon is known as the magnetic catalysis [10, 11]. For large μ the phase boundary has a different nature in comparison with the magnetic catalysis.



(a) $T - \mu$ plot with B fixed. (b) $B - T$ plot with μ fixed.

Figure 2: Critical lines on $T - \mu$ plane and $B - T$ plane. The dashed line represents the first order phase transition while the solid line represents the second order phase transition.

4 Summary

We have investigated the phase structure of NJL model at finite temperature and chemical potential in an external magnetic field. We calculated the effective potential in the leading order of the $1/N_{\rm c}$ expansion by using the Fock-Schwinger proper-time method generalized to the thermal field theory. We deal with the combined effects of the temperature, chemical potential and magnetic field exactly. We found the phase boundary dividing the symmetric phase and broken phase. For small chemical potential the magnetic catalysis is observed. The phase boundary shows more a complex structure for large chemical potential. It may affect the physics in a dense quark matter. We will continue our work further and apply our result to some of dense objects.

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